PROCEEDINGS
JOINT CONVENTION BANDUNG (JCB) 2021
November 23rd – 25th 2021

The Influence of Failure Criteria in Geomechanical Wellbore Stability Modelling

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Abstract

Wellbore instability may cause a significant operational and safety issue if not properly managed. Optimization strategy such as trajectory design based on the safe and stable drilling operation can be done through geomechanical approach to make sure the stability of the wellbore can be maintained. This requires detailed knowledge about the geomechanical parameters including the choice of the criteria to determine the onset of the rock failure. The Mohr-Coulomb failure criterion has been widely used and becomes a standard in petroleum geomechanics. Nevertheless, an appropriate selection of failure criteria plays an important factor in wellbore stability modelling. However, the decision as to which failure criterion is adequate for the problem requires a thorough analysis and investigation. This paper aims to compare several failure criteria commonly used in wellbore stability to examine their behaviour for a given problem. Traditionally, a linear-elastic solution is used to compute the near-wellbore stress through which the stability of the wellbore can be assessed upon inserting them into failure criteria. The piece of rocks around the wellbore will be subjected to a high near-wellbore stress concentration in response to the excavation. For a given hypothetical problem, the stress around the wellbore will be first computed and the onset at which the rock will fail will be obtained based on several failure criteria such as Mohr-Coulomb, Mogi-Coulomb, Modified Lade and Drucker-Prager. The chosen failure criteria are found to have produced a considerable range of variability in terms of predicting the onset of wellbore stability. The result also reveals the significant influence of failure criteria on the stability prediction of a wellbore. Triaxial failure criteria such as Mohr-Coulomb tends to give a more conservative value as compared to the true-triaxial failure criteria such as Mogi-Coulomb. This kind of comparison and assessment support the argument on the importance of selecting adequate failure criteria in wellbore stability modelling to predict the onset of rock shear failure.

Keywords: failure criteria, wellbore stability, stress concentration.

Introduction

Wellbore instability is one of the most serious issues affecting drilling and production operations, required when drilling new wells in order to allocate a safe mud window during drilling operations so that it can be produced. Problems caused by wellbore stability are both time-consuming and costly. Drilling-induced fracture at the borehole wall, due to tensile failure, occurs when the least effective principal stress exceeds the tensile strength of the rocks (Gholami et al., 2014). While borehole breakout occurs if the stress concentration at the borehole wall exceeds the shear strength of the rocks. Inability to mitigate wellbore stability issues can result in a significant operational and safety issue such as stuck pipe, drilling fluid loss, well pack-off (Bahrami et al., 2020; Rahimi, 2014). Addressing such challenges necessitates the use of a geomechanical model that incorporates rock elastic and strength properties, in-situ stress and pore pressure and near-wellbore stress. Geomechanical analysis by using failure criteria can provide an insight regarding the mechanical stability of the borehole wall could potentially reduce Non-Productive Time (NPT). McLean and Addis (1990) argued that the choice of failure criteria is important and has a significant impact on the wellbore stability model. In the prediction of wellbore failure, associated with reservoirs and rock formations in general, several failure criteria are typically considered and evaluated. Determining the appropriate failure criteria for maintaining wellbore stability, on the other hand, can be challenging.

Many researchers have investigated the effect of the choice of failure criteria and are primarily concerned with the quantitative comparison to obtain the best fitting parameters for the various rock failure criteria. Al-Ajmi and Zimmerman (2006) conducted wellbore stability analysis using the Mogi-Coulomb failure criterion and compared the result with Mohr-Coulomb failure criterion. Setiawan and Zimmerman (2019) found that the Mogi-Coulomb criterion predicts lower minimum mud weight by incorporating the strengthening effect of the intermediate principal stress. Meanwhile, Rahimi (2014) mentioned that Drucker-Prager failure criterion estimates the upper bounds of the minimum mud weight for the case under investigation, whereas Mogi-Coulomb and Modified Lade predict results that are lower than Drucker-Prager. These studies were carried out with different parameters and wellbore conditions, hence, it is difficult to justify as to which failure criteria is best representing a general case.

In order to address this, an analysis has been done to investigate the behavior of several failure criteria assuming the same wellbore conditions and in situ geomechanical parameters. This paper investigates four different failure criteria such as Mohr-Coulomb, Mogi-Coulomb, Drucker-Prager, and Modified Lade. These failure criteria are commonly used in wellbore stability modelling. The different radar plots of each failure criteria will be constructed to provide insight regarding the influence of failure criteria on the mechanical wellbore stability.

Near-Wellbore Failure Prediction

In situ stress and near-wellbore stress calculation

To construct a comprehensive wellbore stability model, in-situ and near-field stress of the wellbore needs to be calculated. There are three main steps that need to be done in wellbore stability analysis for an arbitrary oriented wellbore. Firstly, the in-situ stress components need to be transformed from earth coordinate system to wellbore coordinate system. Secondly, near-wellbore stress must be computed depending on the relative wellbore orientation with respect to the in situ stress azimuth. Lastly, the shear failure limit of the rock around the wellbore perimeter is calculated and examined (Setiawan and Zimmerman, 2018).

Traditionally, the Kirsch equation is used to compute the near-wellbore stress. The generalized Kirsch equation proposed by Hiramatsu and Oka (1965) is valid for all wellbore orientations. The equations depends on the in-situ stresses, $\sigma_{x,0}$, $\sigma_{y,0}$, $\sigma_{z,0}$; the wellbore geometry in polar coordinate system, r or R_w (radial distance/wellbore radius), angular direction (θ), and the wellbore pressure (p_w). The stress components of the generalized Kirsch can be written as (Fjaer, 2008):

$$
\begin{aligned}
\sigma_\theta &= \frac{\sigma_{x,0}+\sigma_{y,0}}{2}\left(1+\frac{{R_w}^2}{r^2}\right)-\frac{\sigma_{x,0}-\sigma_{y,0}}{2}\left(1+3\frac{{R_w}^4}{r^4}\right)\cos 2\theta-\tau_{xy,0}\left(1+3\frac{{R_w}^4}{r^4}\right)\sin 2\theta-p_w\left(\frac{{R_w}^2}{r^2}\right)\\
\sigma_r &= \frac{\sigma_{x,0}+\sigma_{y,0}}{2}\left(1-\frac{{R_w}^2}{r^2}\right)+\frac{\sigma_{x,0}-\sigma_{y,0}}{2}\left(1+3\frac{{R_w}^4}{r^4}-4\frac{{R_w}^2}{r^2}\right)\cos 2\theta\\
&\quad +\tau_{xy,0}\left(1+3\frac{{R_w}^4}{r^4}-4\frac{{R_w}^2}{r^2}\right)\sin 2\theta+p_w\left(\frac{{R_w}^2}{r^2}\right)\\
\sigma_z &= \sigma_{z,0}-v\left[2(\sigma_{x,0}-\sigma_{y,0})\frac{{R_w}^2}{r^2}cos2\theta+4\tau_{xy,0}\frac{{R_w}^2}{r^2}sin2\theta\right]\\
\sigma_{r\theta} &= \frac{\sigma_{x,0}-\sigma_{y,0}}{2}\left(1-3\frac{{R_w}^4}{r^4}+2\frac{{R_w}^2}{r^2}\right)\sin 2\theta+\sigma_{xy,0}\left(1-3\frac{{R_w}^4}{r^4}+2\frac{{R_w}^2}{r^2}\right)\cos 2\theta\\
\sigma_{\theta z} &= (-\sigma_{xz,0}\sin\theta+\sigma_{yz,0}\cos\theta)\left(1+\frac{{R_w}^2}{r^2}\right)\\
\sigma_{rz} &= (\sigma_{xz,0}\cos\theta+\sigma_{yz,0}\sin\theta)\left(1-\frac{{R_w}^2}{r^2}\right)
\end{aligned}
\tag{1}
$$

After near-field stress components are obtained, the stability limit of the shear failure can then be calculated using a particular failure criteria. The following section will briefly describe the four different failure criteria, such as Mohr-Coulomb, Mogi-Coulomb, Drucker-Prager, and Modified Lade, considered in this paper.

Mohr-Coulomb Failure Criterion

Mohr-Coulomb is considered to be the most popular and widely used failure criteria for multiple engineering cases, especially in the oil and gas industry. It is one of the conventional triaxial criteria, which only accounts for the minimum and maximum principal stress; the criterion disregards the intermediate principal stress (S_1>S_2=S_3). Specifically, this criterion implies that rock failure occurs toward points where shear stress is greater than the rock's shear strength. The Mohr-Coulomb criterion is represented mathematically as follows (Jaeger, 2007):

$$\sigma_1 = 2S_0\,\frac{\cos\phi}{1-\sin\phi}+\sigma_3\,\frac{1+\sin\phi}{1-\sin\phi} \tag{2}$$

As observed, this failure criteria is popular due to its simple calculation. However, there are situations in which the intermediate principal stress needs to be accounted for to give a more accurate representation of the stress condition. Hence, other options of failure criteria that involves the three principal stress should be considered, i.e. polyaxial stress state (S_1>S_2>S_3).

Mogi-Coulomb Failure Criterion

The importance of the polyaxial stress approach arises from an experiment conducted by Mogi (1971), which confirmed that most fractures occur along the intermediate principal stress direction. It turned out that intermediate principal stress has a strengthening influence on rocks. Mogi implied that $\sigma_{m,2}$, instead of the octahedral normal stress (σ_{oct}), is the mean normal stress that holds against the occurrence of the fracture plane. The basic form of Mogi failure criterion describes that distortional strain energy is proportional to σ_{oct} and increases monotonically with $\sigma_{m,2}$ until failure occurs (Al Ajmi and Zimmerman, 2005). However, it cannot be coupled with Coulomb's strength parameters (c and ϕ) due to being a power-law function (Al Ajmi and Zimmerman, 2006). To address this issue, Al-Ajmi and Zimmerman (2005) proposed a linear version of the function called the Mogi-Coulomb failure criterion, which is represented by the following formula:

$$\tau_{oct} = a + b\sigma_{m,2} \tag{3}$$

In a triaxial stress condition, the Mogi-Coulomb criterion is equivalent to the Mohr-Coulomb criterion. As a result, this failure criterion can be considered as an extension of Mohr-Coulomb (Rahimi, 2014; Al-Ajmi and Zimmerman, 2005). In addition to Mogi-Coulomb, other researchers have also proposed other polyaxial failure criteria.

Drucker-Prager Failure Criterion

The Drucker-Prager criterion originally stems from a failure criterion developed for soil mechanics called the Von Mises criterion, hence why this criterion is also called the Extended Von Mises criterion (Colmenares and Zoback, 2002; Drucker and Prager 1952). In the beginning, Von Mises criterion was discovered through the unempirical means of rotational symmetry of the deviatoric plane. However, Von Mises criterion does not account for mean normal stress, so it defies experimental observation. In order to make it more relevant to rock mechanics, Drucker and Prager expand the failure criterion by involving mean normal stress into its calculation, which would eventually become the Drucker-Prager criterion known as follows:

$$\sqrt{J_2} = k + \alpha J_1 \tag{4}$$

where α is the material constant for internal friction, k is the material constant for cohesion, J_1 is the mean normal stress, and J_2 is the second invariant of deviatoric stress (Colmenares, 2002). Based on its relationship with the Mohr-Coulomb criterion when observed in the principal stress plane, the criterion can be categorized into two versions, Circumscribed and Inscribed. Aside from its different material constant value, they are distinguished by its position on the deviatoric plane relative to Mohr-Coulomb. The circumscribed Drucker Prager encircles the Mohr-Coulomb hexagon and intersects with its outer apices, while the inscribed Drucker Prager is embedded inside the Mohr-Coulomb hexagon. (Rahimi, 2014; Colmenares and Zoback, 2002)

Modified Lade

The Modified Lade criterion was adjusted by Ewy (1999) from its original counterpart, the Lade strength criterion. Initially developed for cohesionless soil with curved yield surfaces, the original Lade was modified to conform to rock mechanics by considering the material constant m as 0 so that the criterion can calculate linear shear strength increase with increasing mean normal stress invariant (Colmenares and Zoback, 2002). In addition, the cohesion parameter S is also introduced. The modification on Lade criterion is defined as follows (Colmenares and Zoback, 2002; Ewy, 1999):

$$\frac{(I_1'')^3}{I_3''} = 27 + \eta \tag{5}$$

where η is the internal friction parameter and I_1'', I_3'' are the first and third stress tensor invariants respectively, defined by the following equation:

$$\begin{aligned} I_1'' &= (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \\ I_3'' &= (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S) \end{aligned} \tag{6}$$

While Modified Lade tends to be inaccurate when tensile stress is considered due to not possessing tension cut off component, this issue does not affect wellbore stability analysis (Rahimi, 2014; Ewy, 1999). This is because tension is not applied to the points of interest which require mud weight.

Result and Discussion

To examine the effects of failure criteria in wellbore stability, radar plots of four different equations have been plotted in Figure 2a. The radar plot represents all possible wellbore orientation from North-East-South-West and from vertical (center of the radar plot) to horizontal wellbore (the outermost ring). The white dot represents the actual wellbore orientation, color-coded contour represents the minimum mud weight to prevent wellbore collapse. An inclined wellbore with 60° deviation towards N 75E with an azimuth of minimum horizontal stress of 45° NE is used in this paper. The rock formation is characterized by a pore pressure of 9 ppg, cohesion of 4.1958 psi, and 31° angle of internal friction. The in situ stress also applies the same values for each failure criteria.

Figures 2b and 2c show the influence of the wellbore inclination and azimuth to the minimum shear failure limit for all failure criteria considered in this paper. The Mohr-Coulomb (MC), Modified Lade (Lade), Drucker-Prager (DP) and Mogi-Coulomb (MgC) are shown as green, blue, red and dark blue, respectively. For all failure criteria, shown in Figure 2b, the trend of minimum shear failure increases as the wellbore becomes more deviated. Nevertheless, they are different in the rate of changes, particularly for Drucker-Prager. Figure 2c shows the profile of the minimum shear failure limit with respect to wellbore azimuth.

When the Mohr-Coulomb criterion is considered, it shows that the color-coded contour produces a unique symmetric pattern where the minimum value of mud weight is towards the NE-SW direction. This condition requires a mud weight of approximately 10.5 ppg EMW. However, an extra mud weight of higher than 11.0 ppg EMW is necessary when the wellbore has an inclination of more than 70° as indicated by the green curve in Figure 2b. From the inclination and azimuth sensitivity plot, it can be seen that the shear failure limit has an uptrend profile with respect to the change in inclination (Figure 2b) while the azimuthal sensitivity of the shear failure limit shows a horizontal trend with "jagged" profile (Figure 2c).

In Figure 2a, the shear failure limit of Mohr-Coulomb has a similar pattern with that of Drucker-Prager; in which the symmetric pattern is also toward the minimum horizontal stress direction. However, it is obvious that the Drucker-Prager is more sensitive for an inclined wellbore of more than 30°. Moreover, according to the Drucker Prager criterion, the predicted shear failure limit is approximately 13.0 ppg EMW to prevent the hole collapse. In contrast, Mogi-Coulomb and Modified Lade resulted in a much smaller predicted minimum shear failure limit. A 60° inclined wellbore only required 9.0 ppg EMW on Mogi-Coulomb, even the highest shear failure limit is only 10.2 ppg EMW. This may be associated with the fact that Mogi-Coulomb criterion accounts for the strengthening effect of the intermediate principal stress, hence, it predicts a lower minimum shear failure limit of the mud weight as compared to Mohr-Coulomb. The Modified Lade is mainly estimating the lowest minimum required mud weight than other rock failure criteria because of overestimation of rock strength (Rahimi, 2014).

For the case considered in this paper, it is interesting to see that for a wellbore inclination of less than 20°, the influence of the intermediate principal stress considered in the Drucker-Prager is not sensitive, i.e. their profile follows that of Mohr-Coulomb. And also, for wellbore with inclination higher than 20°, from all failure criteria considered in this paper, it is obvious that Drucker-Prager gives the most sensitive minimum shear failure limit compared to the other failure criteria.

Figure 2c shows that the minimum shear failure limit of Mogi-Coulomb and Modified Lade seem to be insensitive with the drilling azimuth, i.e., the azimuth profile is rather flat for both failure criteria. Modified Lade predicts the wellbore only required 9.2 ppg EMW for all possible azimuth. However, for Mohr-Coulomb and Drucker-Prager, the azimuthal sensitivity is more fluctuate.

Conclusions

The result discussed in this paper shows the significant influence of failure criteria on the stability prediction of a wellbore. This assessment supports the argument on the influence of failure criteria in wellbore stability modelling to predict the onset of rock shear failure.

In summary, it can be seen that all of the investigated failure criteria produce different profiles of minimum shear failure limit. Mohr-Coulomb and Drucker-Prager produce a more conservative shear failure limit with the difference of the minimum shear failure limit with that of Modified Lade and Mogi-Coulomb, for the case considered in this paper, is approximately 4.0 ppg EMW. Such a difference would considerably affect drilling design. This may not be true for other cases, therefore, to ascertain the influence of the choice of failure criteria for a given case, the wellbore stability analysis should be performed using as many possible failure criteria, whenever possible. Nevertheless, polyaxial failure criteria considered in this paper, i.e. Mogi-Coulomb, Drucker-Prager, Modified Lade, reflect the actual downhole conditions in which the three principal stresses are likely to influence wellbore instability.

Acknowledgements

The authors would like to thank Enerka for the initiative and Ngurah Beni Setiawan for the fruitful discussion.

References

Al-Ajmi, A. M., and Zimmerman, R. W., 2005, International Journal of Rock Mechanics and Mining Sciences, 42(3), 431–439.

Al-Ajmi, A. M., and Zimmerman, R. W, 2006, Proceedings of the IADC/SPE Asia Pacific Drilling Technology Conference, 1, 537-550.

Bahrami, B., Sadatshojaie, A., and Wood, D. A, 2020, Journal of Energy Resources Technology, 142, 8.

Benz, T., & Schwab, R., Journal of Rock Mechanics and Mining Sciences, 45(7), 1176–1186.

Colmenares, L. B., and Zoback, M. D., 2002, International Journal of Rock Mechanics and Mining Sciences, 39(6), 695–729.

Drucker., and Prager., 1952, Quarterly of Applied Mathematics 10(2), 157-165.

Dusseault, M. B., 2011, KSCE Journal of Civil Engineering, 15(4), 669–678.

Ewy, R. T., 1999, SPE Drilling and Completion, 14(2), 85–91.

Fjaer, E., Holt, R. M., Horsrud, P., Raaen, A. M., Risnes, R., 2008, Elsevier.

Gholami, R., Moradzadeh, A., Rasouli, V., and Hanachi, J., 2014, Journal of Rock Mechanics and Geotechnical Engineering, 6(1), 13–25.

Gholami, R., Moradzadeh, A., Rasouli, V., and Hanachi, J, 2014, Journal of Rock Mechanics and Geotechnical Engineering, 6(1), 13–25.

Jaeger, J. C., Cook, N. G. W., Zimmerman, R. W., 2007, Blackwell Publishing.

Han, Y., Liu, C., Phan, D., AlRuwaili, K., and Abousleiman, Y., 2019, SPE Middle East Oil and Gas Show and Conference, MEOS.

Irawan, S., Bataee, M., and Zare, M. R., 2016, Advanced Materials Research, 1133, 624–628.

McLean, M. R., and Addis, M. A., 1990, SPE/IADC Drilling Conference.

Mehranpour, M. H., and Kulatilake, P. H. S. W., 2016, Geomechanics and Geophysics for Geo-Energy and Geo-Resources, 2(4), 203–229.

Rahimi, R., 2014, Master Thesis, Missouri University of Science and Technology.

Setiawan, N. B., and Zimmerman, R. W., 2018a, International Conference on Oil & Gas Engineering and Technology (ICOGET), 13–14.

Setiawan, N. B., and Zimmerman, R. W., 2018b, International Journal of Rock Mechanics and Mining Sciences, 112, 313–322.

Setiawan, N. B., and Zimmerman, R. W, 2019, 53rd U.S. Rock Mechanics/Geomechanics Symposium.

Setiawan, N. B., 2019, Memoir ISPG.

Setiawan, N. B., 2020, thesis, Imperial College London

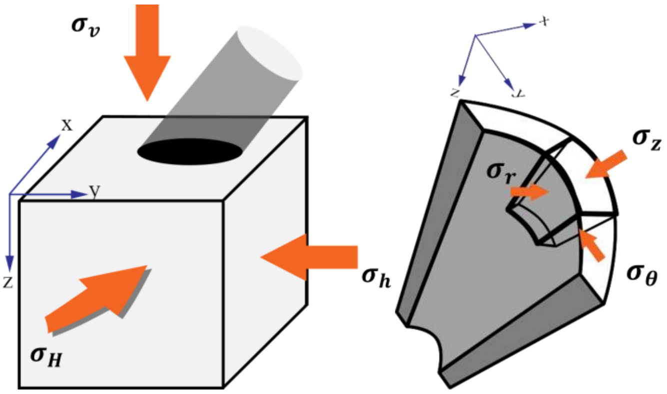

Figure 1: Principal stress and near-wellbore stress ($\sigma_v, \sigma_H, \sigma_h$). The in-situ stress be redistributed around the wellbore to become ($\sigma_\theta, \sigma_r, \sigma_z$) (Modified from Setiawan, 2019).

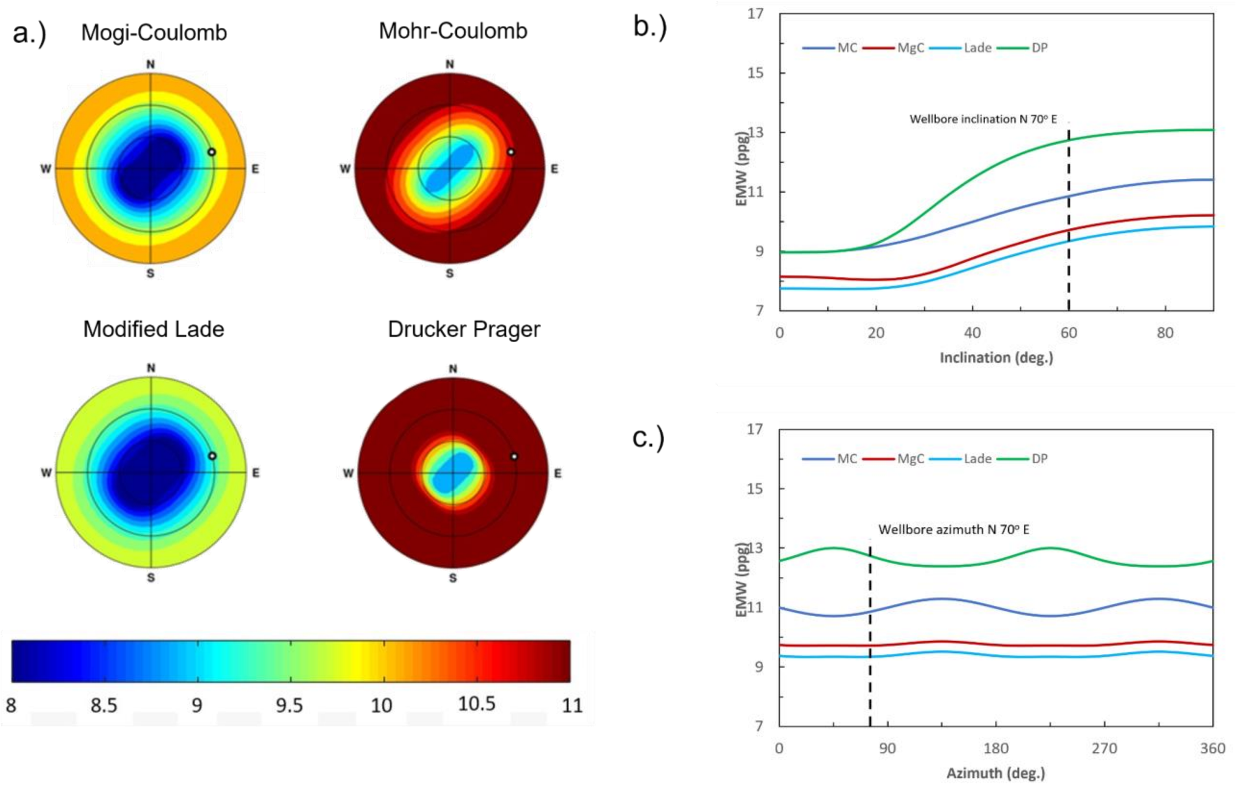

Figure 2: The sensitivity of shear failure limit to avoid wellbore collapse along with the (a) Mohr-Coulomb, Mogi-Coulomb, Modified Lade, and Drucker Prager failure criteria. (b) Inclination and (c) azimuth sensitivity plot.