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The Effect of Poisson's Ratio and Young's Modulus on Fracture Geometry of 2D Model PKN: Case Study of Unconventional Reservoir

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Abstract. Unconventional reservoirs usually have thick reservoir and contain a lot of hydrocarbon. But there is a challenge that must be faced, very tight permeability (<0.001 mD). For this reason, hydraulic fracturing must be applied to create ways. There is no standard equation applied in Indonesia to design the fracture geometry on unconventional reservoir. This paper will discuss about fracture geometry design on unconventional reservoir of Brown Shale, Central Sumatra Basin using the 2D PKN (Perkins-Kern-Nordgren) model. In order to design the fracture geometry, it is necessary to know some geomechanic parameters, such as Poisson's ratio and Young's modulus. Fracture geometry are designed using the 2D PKN (Perkins-Kern-Nordgren) model, the results are fracture length and width. As the results obtained, the sensitivity analysis of Poisson's ratio and Young's modulus are performed to understand the effect of geomechanics on fracture geometry designed. With reservoir height of 153 ft, the fracture length generated is 300.97 ft and average fracture width generated is 0.29 inch. The results of geomechanical sensitivity analysis present the same effect between Poisson's ratio and Young's modulus on the fracture length and fracture width. The lower Poisson's ratio and Young's modulus we get, the fracture length goes shorter, while the fracture width goes wider. The results of the analysis will then generate integrated approaches to estimate the fracture length and fracture width with a certain value of Poisson's ratio and Young's modulus.

Keyword: Fracture Geometry, Poisson's Ratio, Young's Modulus, PKN, Unconventional Reservoir

1 Introduction

Unconventional reservoirs usually have a thick reservoir and contain a lot of hydrocarbons. But there is a challenge that must be faced, very tight permeability (<0.001 mD). For this reason, hydraulic fracturing must be applied to create ways. There is no standard equation applied in Indonesia to design the fracture geometry on unconventional reservoir. This paper will discuss about fracture geometry design using the 2D PKN (Perkins-Kern-Nordgren) model. A study case for this paper was held on unconventional reservoir of Brown Shale, Central Sumatra Basin. As the fracture geometry is designed, a sensitivity analysis method will be carried out to understand the effect of geomechanics. The focus on geomechanics parameters are Poisson's ratio and Young's modulus.

2 Geomechanics

2.1 Compressional-Wave and Shear-Wave Velocities

Castagna et al. (1985) generated the ratio of compressional to shear wave velocity. The Vp/Vs relationship famous established for mudrock line, water-saturated siliciclastic rocks composed primarily of quartz and clay minerals (Castagna et al., 1985).

$$\mathrm{Vs} = 0.862\ \mathrm{Vp} - 1.172 \tag{1}$$

where the compressional and shear wave velocities are in km/s.

2.2 Poisson's Ratio

If a solid body is subjected to an axial tension, it contracts laterally, on the other hand, if it is compressed, the material expands sidewise. So the definition of Poisson's ratio can be stated as the ratio of transverse strain to axial strain induced by unconfined axial deformation (Kumar, 1976). Static Poisson's ratio from uniaxial compression test can be calculated:

$$v = \frac{lateral\ strain}{axial\ strain} = \frac{\varepsilon_x}{\varepsilon_z} \tag{2}$$

Dynamic Poisson's ratio of rock can be determined using empirical equations obtained from the P-wave velocity and S-wave velocity data (Buntoro, et al., 2018). Zoback (2007) generated an empirical equation of Poisson's ratio from sonic log, as shown below:

$$v = \frac{{V_p}^2 - 2{V_s}^2}{2\left({V_p}^2 - {V_s}^2\right)} \tag{3}$$

2.3 Young's Modulus

The Young's modulus was computed from the line resulting from the average of the load-deformation curves obtained during a second test (uniaxial compression test) by the usual stress-strain formula (Heindl & Mong, 1936). Static Young's modulus from uniaxial compression test can be calculated:

$$E = \frac{stress}{strain} = \frac{\sigma}{\varepsilon} \tag{4}$$

Dynamic Young's modulus of rock can be determined using empirical equations obtained from the P-wave velocity and S-wave velocity data (Buntoro, et al., 2018). Fjær et al. (2008) generated an empirical equation of Poisson's ratio from sonic and density log, as shown below:

$$E = \rho \times \mathrm{Vp}^2 \frac{\left(3{V_p}^2 - 4{V_s}^2\right)}{\left({V_p}^2 - {V_s}^2\right)} \tag{5}$$

where E is Young's modulus in GPa.

3 The PKN (Perkins-Kern-Nordgern) Model

The width of vertical fracture was first studied by Khristianovic and Zheltov (1955) with an assumption that the width does not vary in the vertical direction. Thus, a state of plane strain prevails in horizontal planes and the width can be determined as the solution of a plane elasticity problem (Nordgren, 1972). Perkins and Kern (1961) proposed a different approach to determine the fracture width. They considered a vertically limited fracture under the assumption of plane strain in vertical planes perpendicular to the fracture plane. The cross-section of the fracture is found to be elliptical, and the maximum width decreases along the fracture according to a simple formula that contains the fracture length (Nordgren, 1972).

Nordgren (1972) concerned with vertically limited fractures of the type studied by Perkins and Kern (1961) then improved it. He included the effects of fluid loss and fracture volume change in the continuity equation as the fracture length is determined as part of the solution consequently. Probably the most important assumptions are that fracture height is limited vertically and that fracture length is much greater than height (Nordgren, 1972). In Figure 1, Gidley, et al. (1989) showed a schematic of PKN model and modified by Economides and Nolte (2000).

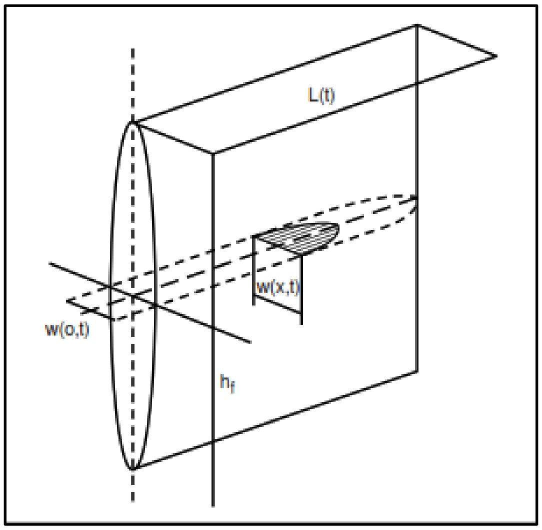

Figure 1. Schematic representation of linearly propagating fracture with laminar fluid flow according to PKN model (Gidley, et al., 1989, modified by Economides & Nolte, 2000)

Gidley, et al. (1989) explained the assumptions in the PKN model for vertical linear fracture propagation, as follows: 1) the fracture has a fixed height, *hf*, independent of fracture length; 2) the fracturing fluid pressure, *p*, is constant in vertical cross-sections perpendicular to the direction of propagation; 3) reservoir rock stiffness, its resistance to deformation under the action of *p*, prevails in the vertical plans, in other words, each vertical cross-section deforms individually and is not hindered by its neighbors; 4) accordingly, in these cross-sections relate height, fluid pressure, and local fracture width as they obtain an elliptic shape with maximum width in the center; 5) the fluid pressure gradient in the propagation or *x* direction is determined by the flow resistance in a narrow, elliptical flow channel; 6) the fluid pressure in the fracture falls off toward the tip or leading edge such that at $x = L$, $p = \sigma_H$ for unspecified reasons.

Economides and Nolte (2000) explained the way to calculate the fracture geometry sequentially from the plane strain modulus, maximum fracture width, average fracture width, turbulent flow correction factor (β), and fracture length (X_f). The plane strain modulus can be expressed as:

$$E' = \frac{E}{(1-v^2)} \tag{6}$$

where E' is plane strain modulus in Pa, E is Young's modulus in Pa, and v is Poisson's ratio. Yang (2012) comprehensively expressed an equation for designing the maximum fracture width, as is below:

$$w_{(0)} = 9.15^{\frac{1}{(2n'+2)}} \times 3.98^{\frac{n'}{(2n'+2)}} \times \left[\frac{1+2.14\,n'}{n'}\right]^{\frac{n'}{(2n'+2)}} \times K'^{\frac{1}{(2n'+2)}} \times \left[\frac{q_i^{n'} \times h_f^{(1-n')} \times X_f}{E'}\right]^{\frac{1}{(2n'+2)}} \tag{7}$$

where w_0 is maximum fracture width in inch, n' is power-law effective index, K' is power-law effective consistency coefficient in lbf-s$^{n'}$/ft^2, q_i is injection rate in bpm, h_f is fracture height in ft, X_f is fracture length in ft.

The average fracture width can be expressed as:

$$\overline{w} = \frac{\pi}{5} w_0 \tag{8}$$

where $\overline{w}$ is average fracture width in inch. The turbulent flow correction factor can be expressed as:

$$\beta = \frac{2C_L\sqrt{\pi t}}{\overline{w}+2S_p} \tag{9}$$

where β is a turbulent flow correction factor, S_p is spurt loss in m^3/m^2, C_L is leak-off coefficient or fluid loss coefficient in m/s$^{1/2}$, and t is injection time in second. The fracture length can be designed as:

$$x_f = \frac{[\overline{w}+2S_p]q_i}{4\pi h_f C_L^2}\left[\exp(\beta^2)\, erfc(\beta) + \frac{2\beta}{\sqrt{\pi}} - 1\right] \tag{10}$$

4 Result and Discussion

Calculation of fracture geometry was carried out using PKN model based on the assumptions used. Data needed was collected from Well FW of sweet spot in Brown Shale, Central Sumatra Basin as an unconventional reservoir. The fracture width and length design have been obtained from 10 iterations with final error percentage of -1.28×10^{-10}%. The result is valid according to the assumption used that the percentage of error must be lower than 0.0001 and greater than -0.0001. The results of fracture geometry are 0.35 inches of average fracture width and 300.97 ft of fracture length.

Explicitly, geomechanics parameters are present in the equation to calculate the fracture width and length. This paper is intended to understand the effect of geomechanics changing on fracture geometry designed using PKN model. The geomechanics parameters being focused in the discussion are Poisson's ratio and Young's modulus. Sensitivity analysis Poisson's ratio and Young's modulus on fracture geometry is used to understand the effect. As the sensitivity of Poisson's ratio is analyzing, the Young's modulus is being a constant variable. It is likewise as the sensitivity of Young's modulus is analyzing, the Poisson's ratio is being a constant variable.

The sensitivity analysis was carried out using 26 data of Poisson's ratio and Young's modulus. Both of these sensitivity analysis were done from calculations based on several iterations to obtain valid results with an average of 10 iterations and final error percentage is lower than 1.5×10^{-10}%. A cross plot of fracture geometry and geomechanics parameters were generated to understand the correlation. Figure 2, a cross plot of fracture width/length and Poisson's ratio are shown. And Figure 3, a cross plot of fracture width/length and Young's modulus are shown.

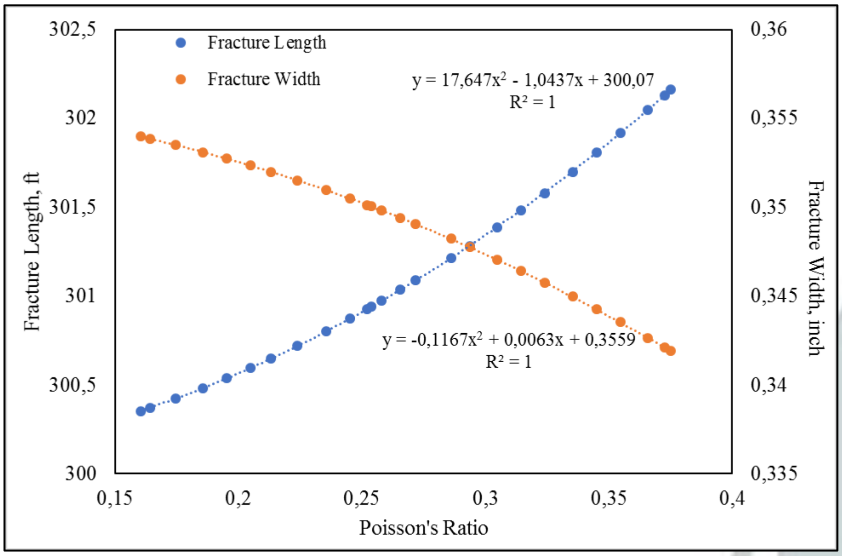

Figure 2. Crossplot of fracture width/length and Poisson's ratio

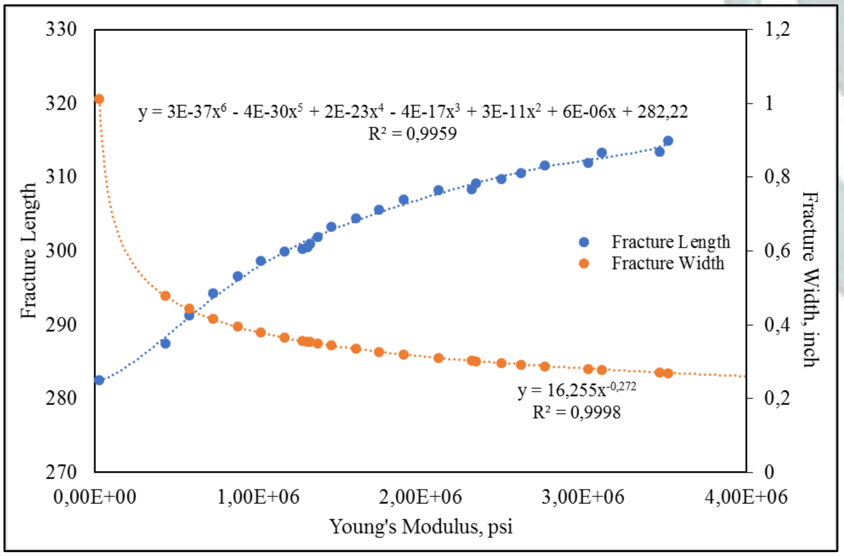

Figure 3. Crossplot of fracture width/length and Young's modulus

Based on Figure 2, there is a relationship between fracture width/length and Poisson's ratio using polynomial regression. The lower Poisson's ratio obtained, the fracture length goes shorter, while the fracture width goes wider. There are also empirical relationships, the way easier to obtain fracture width and length from Poisson's ratio, as shown below:

$$X_f = 17.647\, \upsilon^2 - 1.0437\, \upsilon + 300.07 \quad (11)$$

$$\overline{w} = -0.1167\, \upsilon^2 + 0.0063\, \upsilon + 0.3559 \quad (12)$$

Both of these empirical relationships show coefficient correlation of 1. It means the relationship between fracture width/length and Poisson's ratio is valid to apply in the study area.

Based on Figure 3, there is a relationship between fracture width/length and Young's modulus polynomial and power regression. The lower Young's modulus obtained, the fracture length goes shorter, while the fracture width goes wider. There are also empirical relationships, the way easier to obtain fracture width and length from Young's modulus, as shown below:

$$X_f = 3.10^{-37}\, E^6 - 4.10^{-30}\, E^5 + 2.10^{-23}\, E^4 - 4.10^{-17}\, E^3 + 3.10^{-11}\, E^2 + 6.10^{-06}\, E + 282.22 \quad (13)$$

$$\overline{w} = 16.255\, E^{-0.272} \quad (14)$$

Both of these empirical relationships show the coefficient correlation respectively of 0.9959 and 0.9998. It means the relationship between fracture width/length and Young's modulus is valid to apply in the study area.

5 Conclusion

The study of geomechanics effect on fracture geometry of PKN model has been carried out using sensitivity analysis. The analysis was focused on Poisson's ratio and Young's modulus from Well FW of sweet spot in Brown Shale, Central Sumatra Basin as an unconventional reservoir. Base results of fracture geometry are 0.35 inches of average fracture width and 300.97 ft of fracture length, with reservoir height of 153 ft. And the results of sensitivity analysis present the same effect between Poisson's ratio and Young's modulus on the fracture length and fracture width. The lower Poisson's ratio or Young's modulus obtained, the fracture length goes shorter, while the fracture width goes wider. Integrated approaches of empirical relationship are also generated to estimate the fracture length and fracture width with a certain value of Poisson's ratio and Young's modulus.

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